MAT137 Lecture 1

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Introduction

- **MAT137** Calculus!
  Lec 0401, Monday 15:00-16:00, Thursday 17:00-19:00
  MP137
- **Instructor:** Huan Vo
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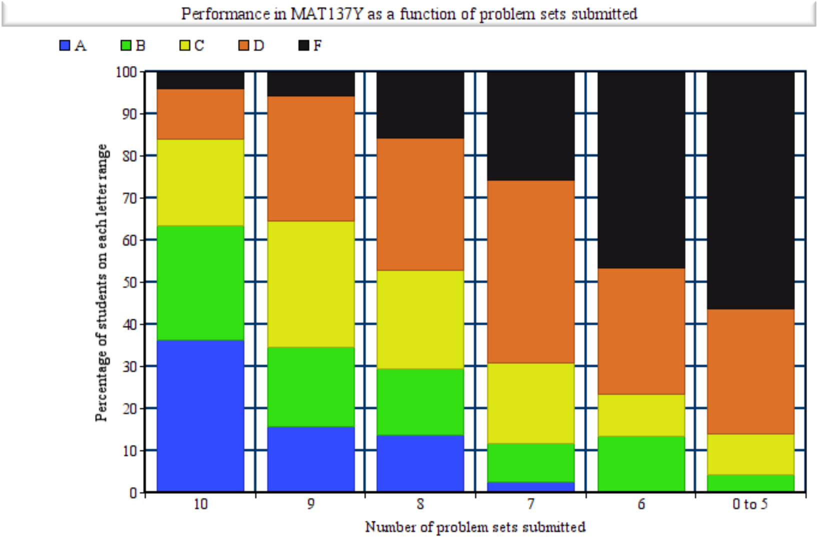
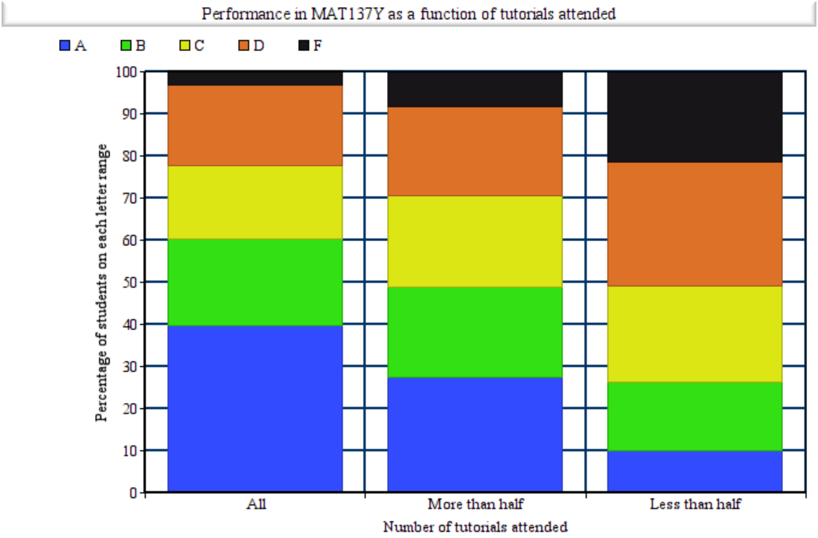
- **Office Hours:** Monday 13:00-15:00
- **Email:** vohuan@math.toronto.edu
- **Course Website:** http://uoft.me/MAT137
  - Read the course outline
  - Online forum: piazza
  - Precalculus review: http://uoft.me/precalc
  - Sign up for tutorials

Some Statistics

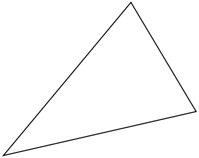


Some Statistics



Why do we need proofs?

Recall the following basic fact of Euclidean geometry:



The sum of the internal angles of a triangle on the plane equals 180 degrees.

Why do we need proofs?

Efforts to prove the above fact from the other four axioms of Euclidean geometry led to the discovery of non-Euclidean geometries. The most famous one is called **hyperbolic geometry**

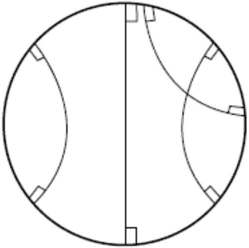
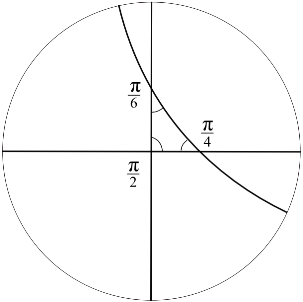


Figure: The Poincare Disk model of hyperbolic geometry

Why do we need proofs?

In hyperbolic geometry, the sum of the internal angles of a triangle is **less** than 180 degrees.



Hyperbolic geometry is a very active area of mathematical research.

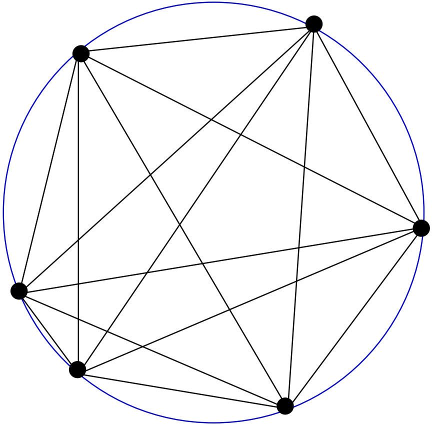
Why do we need proofs?

Let us look at another problem:

Given n distinct points on a circle. Join any two points by a chord. What is the maximum number of regions that the interior of the circle is divided?

Why do we need proofs?

Let us look at a few simple cases



n	number of regions
1	
2	
3	
4	
5	
6	
n	???

Why do we need proofs?

The maximum number of regions for n points is

$$1 + \binom{n}{2} + \binom{n}{4},$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Check your answers!

Game of Thrones

Which of the following statements are equivalent to the statement

"No two students in this class do not like Game of Thrones."?

1. "All student in this class, except at most one, like Game of Thrones."
2. "At least two students in this class do not like Game of Thrones."
3. " If I choose any pair of students in this class and one of them does not like Game of Thrones, then the other one does."
4. "There are at least two students in this class who like Game of Thrones."

Which statements are equivalent to the opposite of the above statement?

Negation

Write the negation of this statement without using any negative words ("no", "not", "none", etc.):

"Every page in this book contains at least one word whose first and last letters both come alphabetically before L."

Answer: "There is a page in this book on which every word either has the first letter or the last letter coming alphabetically after L."

Sets

Describe the following sets in the simplest terms that you can

1. $(1, 3] \cup (2, 4]$
2. $(1, 3] \cap (2, 4]$
3. $(1, -1)$
4. $[2, 2]$
5. $(2, 2)$
6. $A = \{x \in \mathbb{R} : x^2 < 6\}$
7. $B = \{x \in \mathbb{Z} : x^2 < 6\}$
8. $C = \{x \in \mathbb{N} : x^2 < 6\}$

Some more Notations for Sets

Given two subsets A and B of some set S, we define

- $A \setminus B = \{x \in A : x \notin B\}$. We call usually call this A **minus** B or A **without** B. In words, $A \setminus B$ is the set of things in A that are not in B.
- $A\Delta B = (A \setminus B) \cup (B \setminus A)$. We call this the **symmetric difference between** A **and** B. In words, it is the set of all things that are in A and B but not both.

Exercise: Convince yourself that

$$A\Delta B = (A \cup B) \setminus (A \cap B).$$

Some Set Problems

Define the following two sets

- $A = \{\text{female studens in this class}\}$
- $B = \{\text{students sitting in the first two rows}\}$

What are $A \setminus B$, $B \setminus A$, $A \Delta B$?

Some Set Problems

- A **rational** number is a real number of the form p/q where $p, q \in \mathbb{Z}$ and $q \neq 0$.
- A real number which is not rational is called **irrational**.
- We denote the set of positive real numbers by $\mathbb{R}_{>0}$ and the set of negative real numbers by $\mathbb{R}_{<0}$.

Let A be the set of negative irrational numbers and positive rational numbers. Describe A using set operations.

Set Defined with Quantifiers

Describe the following sets in the simplest terms that you can

1. $A = \{x \in \mathbb{R} : \forall y \in [1, 2], \quad x < y\}$.
2. $B = \{x \in \mathbb{R} : \exists y \in [1, 2] \quad \text{such that} \quad x < y\}$.
3. $C = \{x \in [1, 2] : \forall y \in [1, 2], \quad x < y\}$.
4. $D = \{x \in [1, 2] : \exists y \in [1, 2] \quad \text{such that} \quad x < y\}$.
5. $E = \{x \in [1, 2] : \exists y \in \mathbb{R} \quad \text{such that} \quad x < y\}$.
6. $F = \{x \in [1, 2] : \forall y \in \mathbb{R}, \quad x < y\}$.

Next Class: Monday Sept 11

Watch videos 4, 5, 6 in Playlist 1.