MAT137 Lecture 10

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Test 1

- Test 1 will take place on Friday, October 20.
- If you have a conflict, you need to send an email to alfonso@math.toronto.edu by Friday, October 13.

Agenda

- Extreme Value Theorem (EVT).
- Intermediate Value Theorem (IVT).
- Definition of Derivative

Extreme Value Theorem (EVT)

State the extreme value theorem.

Extreme Value Theorem (EVT)

Find a function f on $[0, 1]$ that has no maximum and no minimum.

Extreme Value Theorem (EVT)

Find a *continuous* function f on $(0, 1]$ that has a maximum but no minimum.

Extreme Value Theorem (EVT)

Find a *continuous* function f on $(0, 1]$ that has no maximum and no minimum.

Intermediate Value Theorem (IVT)

A function f is defined on the interval $[a, b]$. Which of the following statements are true?

1. If $f(a) > 0$ and $f(b) < 0$, then there must exist at least one number c in (a, b) for which $f(c) = 0$.
2. If f is continuous on $[a, b]$ with $f(a) < 0$ and $f(b) > 0$, then there must exist at least one number c in (a, b) for which $f(c) = 0$.
3. If f is continuous on (a, b) with $f(a) > 0$ and $f(b) < 0$, then there must exist at least one number c in (a, b), for which $f(c) = 0$.
4. If f is continuous on $[a, b]$ with $f(c) = 0$ for some number c in (a, b), then $f(a)$ and $f(b)$ have opposite signs.

Intermediate Value Theorem (IVT)

Show that the equation

$$\sin x + 2\cos x - x^2 = 0.$$

has a solution in $[0, \pi/2]$.

Intermediate Value Theorem (IVT)

Suppose f is continuous on $[1, 5]$ and the only solutions of the equation $f(x) = 6$ are $x = 1$ and $x = 4$. If $f(2) = 8$, explain why $f(3) > 6$.

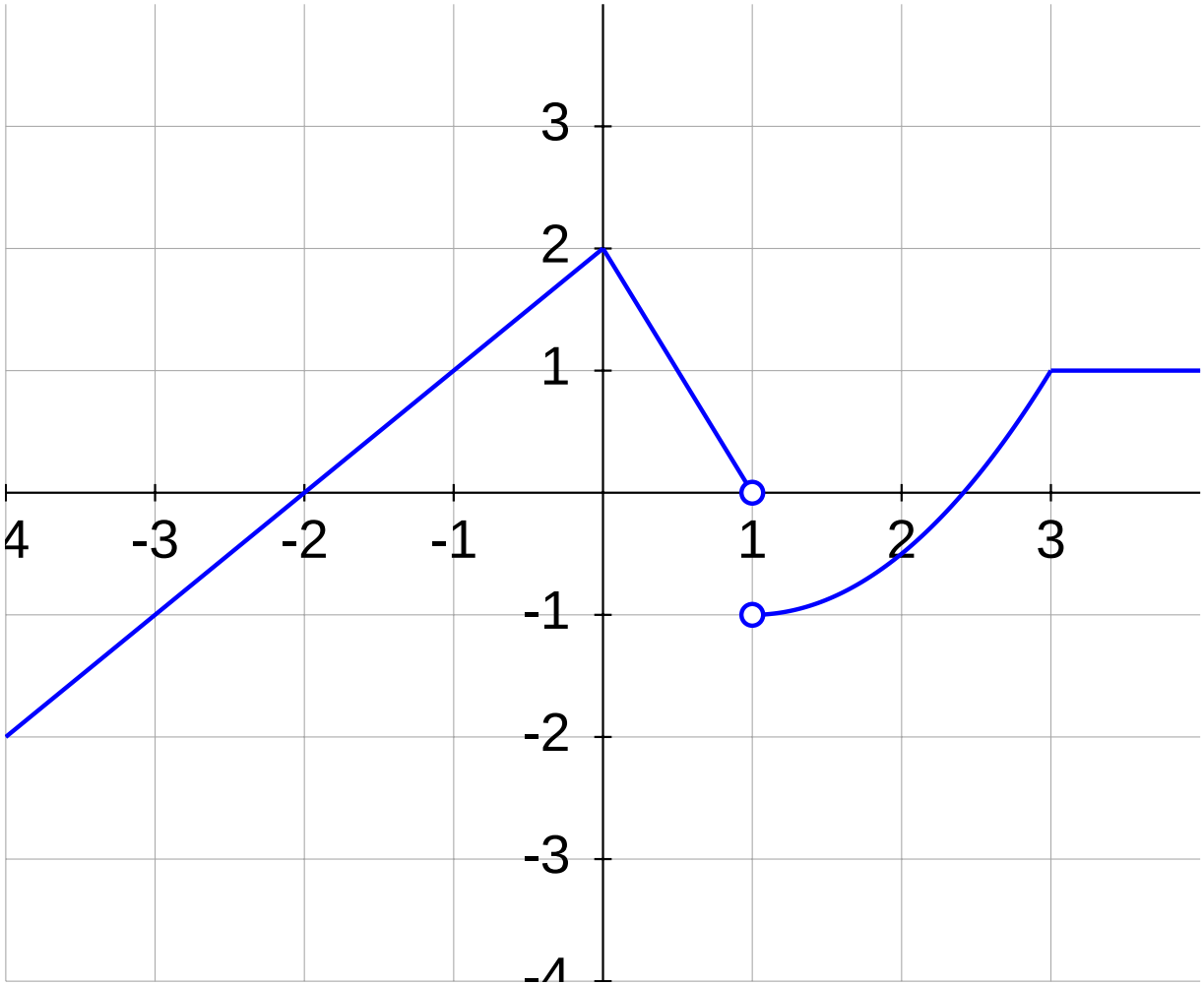
Derivative

Which are the correct definition of $f'(a)$?

1. $\dfrac{f(x)-f(a)}{x-a}$
2. $\lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x)-f(a)}{\Delta x}$
3. $\lim\limits_{h \to 0} \dfrac{f(a+h)-f(a)}{h}$
4. $\lim\limits_{x \to a} \dfrac{f(x)-f(a)}{x-a}$
5. the slope of the tangent line to the graph of f at $(a, f(a))$
6. $\lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$

Graph

The graph of f is shown below



At which numbers c is f not differentiable?

Graph

Problem: Suppose that $f(x)$ is the derivative of some function $g(x)$, can you sketch the function $g(x)$?

Compute Derivative from Definition

Using the definition of derivative, find $f'(2)$, where

$$f(x) = \sqrt{6 - x}.$$

Tangent Line

Write an equation for the tangent line at $(c, f(c))$, where

$$f(x) = \frac{1}{x^2}, \quad c = -2.$$

Differentiation Rules

Find $f'(0)$ given that $h(0) = 3$ and $h'(0) = 2$, where

$$f(x) = h(x) + \frac{x}{h(x)}.$$

Next Class: Monday Oct 16

Watch videos 5, 6, 7 in Playlist 3.