MAT137 Lecture 12

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Halloween Costume

The amount of money collected is 64.80, which exceeds the target donation goal set at 50, so I have to wear a Halloween costume on Thursday Oct 26. Yay...

Agenda

- Higher-order derivatives
- The chain rule
- Derivatives of trigonometric functions

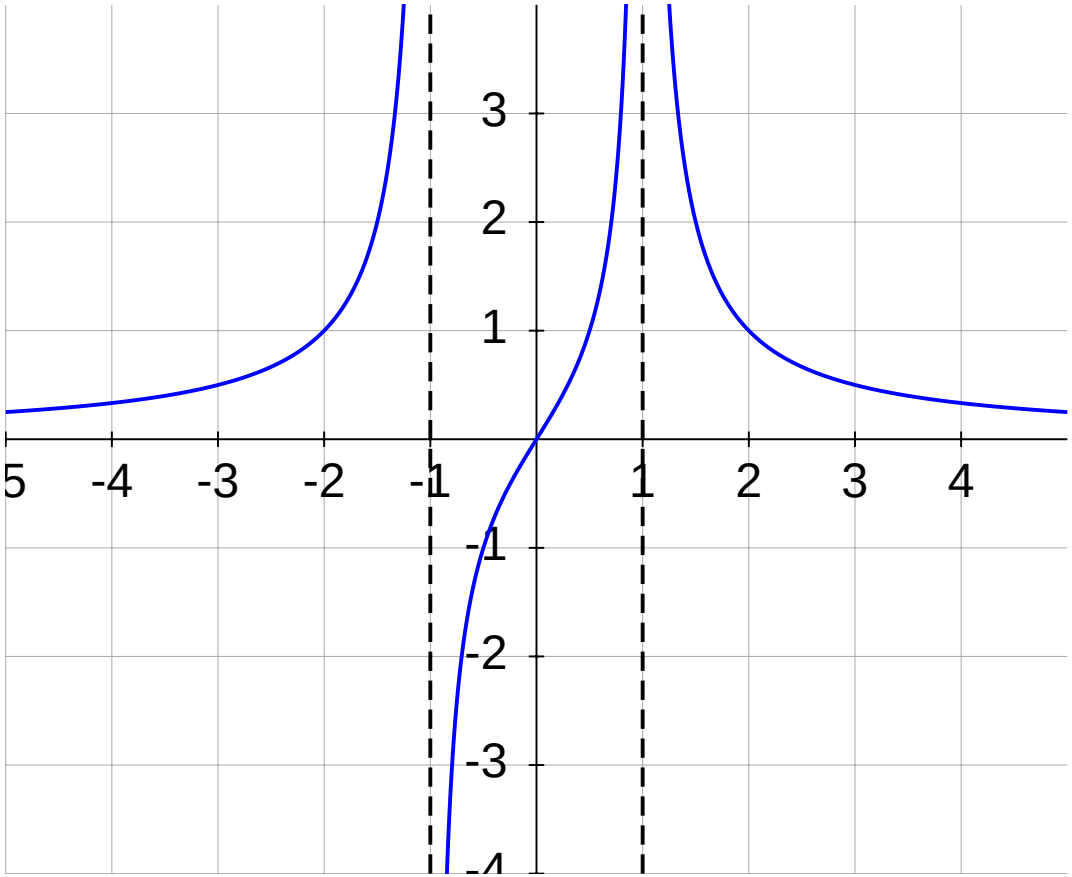
Quotient Rule

Suppose that f and g are differentiable at a and $g(a) \neq 0$. Show that

$$\left(\frac{f}{g}\right)'(a) = \frac{f'(a)g(a) - g'(a)f(a)}{[g(a)]^2}.$$

Graphs of Derivatives

Consider the graph of a function f given as follows



(a) Sketch the graph of a *continuous* function g whose derivative is f.

(b) Sketch the graph of a *discontinuous* function g whose derivative is f.

Higher-order derivatives

Choose the correct definition of $f''(a)$

(a) $\left.\dfrac{\mathrm{d}^2 f}{\mathrm{d}x^2}\right|_{x=a}$

(b) $f'(f'(a))$

(c) $\dfrac{\mathrm{d}}{\mathrm{d}x}[f'(a)]$

(d) $\left.\dfrac{\mathrm{d}f'}{\mathrm{d}x}\right|_{x=a}$

(e) $\lim\limits_{h\to 0}\dfrac{f'(a+h)-f'(a)}{h}$

(f) $\lim\limits_{h\to 0}\dfrac{f'(a)-f'(a-h)}{h}$

Higher-order derivatives

Set

$$g(x) = \begin{cases} x^3, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

(a) Find $g'(0)$ and $g''(0)$.

(b) Determine $g'(x)$ and $g''(x)$ for all other x.

(c) Show that $g'''(0)$ does not exist.

(d) Sketch the graphs of g, g', g''.

Higher-order derivatives

Let p be an arbitrary polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0, \ a_n \neq 0.$$

(a) Find $\dfrac{\mathrm{d}}{\mathrm{d}x^n}[p(x)]$.

(b) What is $\dfrac{\mathrm{d}}{\mathrm{d}x^k}[p(x)]$ for $k > n$?

The chain rule

Which of the following give us $(f \circ g)'(a)$?

(a) $f'(g(a))$

(b) $g'(f(a))$

(c) $f'(g(a))g'(a)$

(d) $f'(g'(a))$

(e) $\left.\dfrac{\mathrm{d}f}{\mathrm{d}u}\right|_{u=g(a)} \left.\dfrac{\mathrm{d}g}{\mathrm{d}x}\right|_{x=a}$

(f) $\lim\limits_{h\to 0} \dfrac{f(g(a)+h)-f(g(a))}{h}$

(g) $\lim\limits_{h\to 0} \dfrac{f(g(a+h))-f(g(a))}{h}$

The chain rule

Compute

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\left(\frac{2x-1}{x+4}\right)^{2017}+\left(x^2+\frac{1}{x^3}\right)^{2019}\right]$$

The chain rule

Suppose that f is differentiable at a, h is differentiable at $f(a)$, g is differentiable at $h(f(a))$, find a formula for

$$(g \circ h \circ f)'(a).$$

The chain rule

Let

$$y = \frac{1+s}{1-s},\ s = t - \frac{1}{t},\ t = \sqrt{x}.$$

Find $\mathrm{d}y/\mathrm{d}x$ at $x = 2$.

Derivative of trigonometric functions

Prove the following formula

(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\tan x = \sec^2 x$

(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\csc^2 x$

(c) $\dfrac{\mathrm{d}}{\mathrm{d}x}(\sec x) = \sec x \tan x$

(d) $\dfrac{\mathrm{d}}{\mathrm{d}x}(\csc x) = -\csc x \cot x$

Derivative of trigonometric functions

Compute the following limit

$$\lim_{x \to \pi/4} \frac{\tan x - 1}{x - \pi/4}$$

Derivative of trigonometric functions

Compute the derivative

$$\tan\left[\sin(x^2+\cos(x^{2019}))^{2017}\right]$$

Next Class: Monday Oct 23

Watch videos 12, 13, 14, 15 in Playlist 3.