MAT137 Lecture 14

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Agenda

Exponentials and logarithms.

Definition of e

Definition

We define e to be the number such that

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1.$$

Exponential Functions

Differentiate the function

$$f(x) = e^{\tan(x^2-1)}.$$

Evaluate the limit

$$\lim_{x\to 1} \frac{e^{\tan(x^2-1)} - 1}{x - 1}.$$

Exponential Function

Without using the power rule, prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^{\pi}) = \pi x^{\pi-1}, \quad x > 0.$$

Derivative of Exponential Function

Find y' if $e^{x/y} = x - y$.

Properties of logarithmic functions

Let $a > 0$ and $a \neq 1$, $x > 0$, $y > 0$ and r is any real number, show that

(a) $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

(b) $\log_a(x^r) = r \log_a x$

(c) $\log_a x = \dfrac{\ln x}{\ln a}$

Logarithmic Functions

Evaluate the limit

$$\lim_{x\to\infty}\left[\ln\left(2+x^{2017}\right)-\ln\left(\sin(x^{2016})+3x^{2017}\right)\right].$$

(a) 0

(b) D.N.E.

(c) ∞

(d) $-\ln 3$

(e) $\ln 2$

Logarithmic Functions

Evaluate the limit

$$\lim_{x \to 1} \frac{\ln\left(\ln e^{e^{x^{2017}}}\right) - 1}{x - 1}.$$

Derivative of Logarithmic Functions

Find $f'(x)$ if $f(x) = \ln|x|$. Sketch the graphs of f and f'.

Derivative of Logarithmic Functions

Find $f'(x)$ if

$$f(x) = \log_{x^2+2}\left(\frac{1}{\sqrt[2017]{x^{2016}+1}}\right).$$

Logarithmic Differentiation

Let $f(x) = x^{\sqrt{x}} + x^x$. Find $f'(x)$.

Logarithmic Differentiation

Find y' if $x^y = y^x$.

Next Class: Monday Oct 30

Watch videos 4.1, 4.2, 4.3, 4.4 in Playlist 4.