MAT137 Lecture 15

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Agenda

Inverse Functions.

The inverse of a function

Definition

Let $f\colon A \to B$ be a one-to-one and onto function. Then the *inverse* of f is the function $f^{-1} : B \to A$ given by

$$f^{-1}(y) = x \quad \iff \quad f(x) = y.$$

Suppose a function $f : A \to B$ has the inverse $f^{-1} : B \to A$, show that

(a) $f(f^{-1}(y)) = y, \quad \forall y \in B.$

(b) $f^{-1}(f(x)) = x, \quad \forall x \in A.$

Warning: f^{-1} is NOT the same as $1/f$.

The inverse of a function

(a) If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$.

(b) If $h(x) = x + \sqrt{x}$, find $h^{-1}(6)$.

The inverse of a function

Find f^{-1} for each of the following f

(a) $f(x) = x^3 + 1$.

(b) $f(x) = \begin{cases} x, & x \text{ is rational,} \\ -x, & x \text{ is irrational} \end{cases}$

(c) $f(x) = x^2 - x, \quad x \leq \frac{1}{2}$

(d) $f(x) = \dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}$

Injective functions

Let $f : A \to B$ be a function and suppose there exists a function g such that $g(f(x)) = x$ for all $x \in A$. Show that f is injective.

Surjective functions

Let $f : A \to B$ be a function and suppose there exists a function h such that $f(h(y)) = y$ for all $y \in B$. Show that f is surjective.

The inverse of a function

Consider the following function

$$f(x) = \begin{cases} -x^2, & x \geq 0, \\ 1 - x^3, & x < 0. \end{cases}$$

(a) Show that f is injective, a.k.a. one-to-one.

(b) Describe the range B of f.

(c) Write down $f^{-1} : B \to \mathbb{R}$.

The inverse of a function

For constants a, b, c, d such that $a \neq 0$ and $c \neq 0$, consider the function

$$f(x) = \frac{ax + b}{cx + d}, \quad x \neq -d/c.$$

Suppose that $ad - bc \neq 0$, show that f is one-to-one and write down f^{-1}. What happens when $ad - bc = 0$?

Next Class: Thu November 2

Watch videos 4.5, 4.6, 4.7, 4.8 in Playlist 4.