MAT137 Lecture 16

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Agenda

- Continuity and Differentiability of Inverse Functions.
- Inverse Trigonometric Functions.
- Relate Rates.

Derivatives of Inverse Functions

Recall that if $f(b) = a$ and $f'(b) \neq 0$, then f has an inverse f^{-1} near a and b which is differentiable and

$$\boxed{(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))} = \frac{1}{f'(b)}}$$

Find $(f^{-1})'(a)$, where

(a) $f(x) = 3 + x^2 + \tan(\pi x/2)$, $-1 < x < 1$, $a = 3$.

(b) $f(x) = \sqrt{x^3 + 4x + 4}$, $a = 3$.

Derivatives of Inverse Functions

Suppose f^{-1} is the inverse function of a differentiable function f and let $G(x) = 1/f^{-1}(x)$. If $f(3) = 2$ and $f'(3) = 1/9$, what is $G'(2)$?

(a) $-1/89$

(b) $-1/9$

(c) -1

(d) D.N.E.

(e) 9

Differentiability of Inverse Functions

Give an example of a differentiable function whose inverse is not differentiable.

Graphs

Sketch the graph of a function g that satisfies all the following properties

(a) The domain of g is $\mathbb{R}$.

(b) g is continuous everywhere except at -2.

(c) g is differentiable everywhere except at -2 and 1.

(d) g is one-to-one, a.k.a injective.

(e) $(g^{-1})'(-4) = 2$.

(f) g^{-1} has a horizontal tangent line at 2.

Inverse trigonometric functions

State the domain and range of the following functions

(a) $f(x) = \arcsin(x^2 - 1)$.

(b) $g(x) = \arccos(2 - x^3)$.

Inverse trigonometric functions

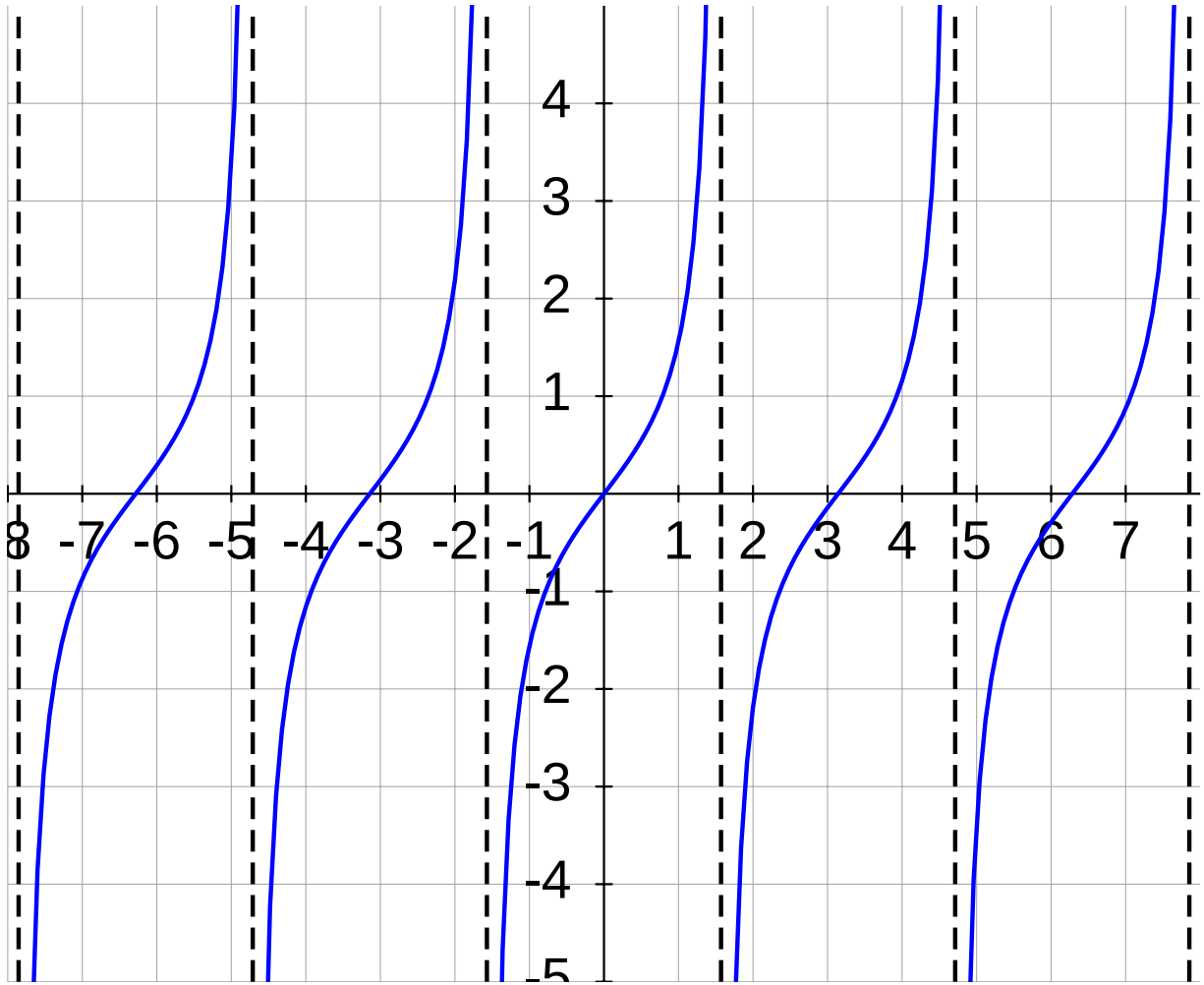
Compute

(a) $\arcsin(\sin 6)$

(b) $\arcsin(\sin 10)$

The inverse tangent function

Recall that the graph of $f(x) = \tan x$ is



The inverse tangent function

The function $f(x) = \tan(x)$ is clearly not one-to-one on $\mathbb{R}$. To obtain a one-to-one function we restrict the domain to $(-\pi/2, \pi/2)$.

Definition

The *inverse tangent function* $\arctan(x)$ is defined as the inverse of $f(x) = \tan x$, $-\pi/2 < x < \pi/2$.

More concisely,

$$\boxed{\arctan(y) = x \iff \tan x = y \text{ and } -\frac{\pi}{2} < x < \frac{\pi}{2}}$$

The inverse tangent function

- Graph the function $f(x) = \arctan(x)$.
- Find the derivative of f.
- Compute $\arctan(\tan 10)$.

Inverse trigonometric functions

Find formulas for the following expressions, using rational functions and roots (if necessary). Write the values of x for which the formula is valid.

(a) $\sin(\arccos x)$

(b) $\sin(\arctan x)$

Related Rates

The general strategy to solve related rate problems is

- Read the problem carefully.
- Draw a diagram if possible.
- Introduce appropriate notations.
- Express the given information and the required rate in terms of derivatives.
- Write an equation that relates the various quantities of the problem.
- Apply the chain rule.
- Solve for the unknown rate.

Related Rates

Problem 1

A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8 m from the dock?

Solution: See here.

Related Rates

Problem 2

The radius of a sphere is increasing at a rate of 4 mm/s. How fast is the volume increasing when the diameter is 80 mm?

Solution: See here.

Related Rates

Problem 3

Water is leaking out of an inverted conical tank at a rate of 10,000 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 8 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.

Solution: See here.

Next Class: Monday November 13

Watch videos 5.1, 5.2, 5.3, 5.4 in Playlist 5.