MAT137 Lecture 17

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Agenda

- Local Extrema.
- The Local Extreme Value Theorem.

Recap

Definitions

Let f be a function with domain some interval I.
Let $c \in I$.

- We say that f has an (**absolute**) **maximum** at c when

$$\forall\ x \in I, \quad f(x) \leq f(c).$$

  In this case we call $f(c)$ the **maximum value** of f. Note that f can have several different maximum points but at most one maximum value.

- We say that f has a **local maximum** at c if c is an interior point and

$$\exists \delta > 0 \quad \text{such that} \quad \forall\ x \in I,\ |x - c| < \delta \implies f(x) \leq f(c).$$

Exercise: State the definitions for minimum, minimum value, local minimum.

Recap

Theorem (The Local Extreme Value Theorem)

Let f be a function with domain some interval I and $c \in I$.
IF

- *f has a local extremum at c, and*
- *c is an interior point to I (not an endpoint)*

THEN

- *$f'(c) = 0$ or D.N.E.*

We say that c is a **critical point** of f if c is an interior point and $f'(c) = 0$ or D.N.E.

Warning: The converse of the local extreme value theorem is NOT true. Namely, a critical point may not be a local extremum.

Graph sketching

For each of the following conditions, sketch the graph of a function f defined on $[-1, 1]$ that satisfies it.

(a) f has a maximum but no local maximum.

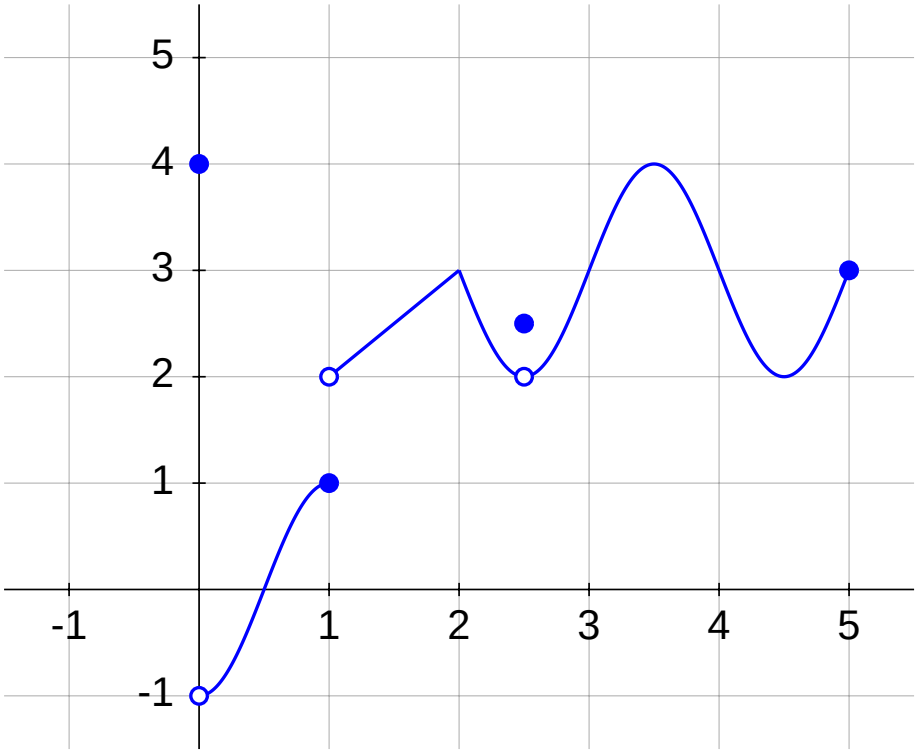
(b) f has a local maximum but has no maximum.

(c) f has a local maximum at 0 but is not continuous at 0.

(d) f has one local minimum, three local maxima, six critical points.

Critical points

Consider a function f defined on $[0, 5]$ whose graph is given by



1. Find all the critical points of f.
2. State whether these points are local max, local min, or neither.
3. Does f have a maximum or a minimum?
4. What are the maximum value and the minimum value?

Finding the extrema of a function

The closed interval method

To find the maximum and minimum values of a *continuous* function f on a closed interval $[a, b]$:

1. Find the values of f at the critical points of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the maximum value, the smallest of these values is the minimum value.

Find the maximum and minimum values of f on the given interval

$$f(x) = \frac{x}{x^2 - x + 1}, \quad x \in [0, 3].$$

Finding the extrema of a function

Show that the following function

$$f(x) = x^9 + 9x^5 + 36x + 7$$

has neither a local maximum nor a local minimum. Does it have a maximum or a minimum?

Finding the extrema of a function

Find the local extrema of the following functions

(a) $f(x) = |x-1| + |x-2|, \quad x \in [0,3]$.

(b) $f(x) = \begin{cases} x, & x \text{ is rational,} \\ 0, & x \text{ is irrational.} \end{cases}$

Next Class: Thursday November 16

Watch videos 5.5, 5.6, 5.7, 5.8, 5.9 in Playlist 5.