MAT137 Lecture 19

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Agenda

Monotonicity of functions.

Recap

Definition

Let f be a function defined on an interval I.

- f is **increasing on** I when

$$\forall\, x_1, x_2 \in I, \quad x_1 < x_2 \implies f(x_1) < f(x_2).$$

- f is **non-decreasing on** I when

$$\forall\, x_1, x_2 \in I, \quad x_1 < x_2 \implies f(x_1) \leq f(x_2).$$

Exercise. State the analogous definitions for **decreasing** and **non-increasing**.

Monotonicity of functions

Prove the following theorem

Theorem

Let $a < b$. Let f be a function defined on (a, b).
IF

$$\forall\ x \in (a, b),\ \ f'(x) > 0$$

THEN f is increasing on (a, b).

Hint. Use the MVT.

Monotonicity of functions

Prove the following theorem

Theorem

Let $a < b$*. Let* f *be a function defined on* $[a, b]$*.*
IF

- $\forall\ x \in (a, b),\ \ f'(x) > 0$*, and*
- f *is continuous on* $[a, b]$

THEN f *is increasing on* $[a, b]$*.*

Hint. Use the MVT. Why do we need the continuity assumption?

Question

Suppose that $f'(x) > 0$ on (a, b) except at finitely many points where $f'(x) = 0$. Is f still increasing? How about when $f'(x) > 0$ on (a, b) except at finitely many points where $f'(x)$ D.N.E.?

Intervals of increasing/decreasing

Find the intervals on which f is increasing or decreasing, where

$$f(x) = 5x^{2/3} - 2x^{5/3}.$$

Proving inequalities using increasing/decreasing

Prove that, for all $x > 1$,

$$2\sqrt{x} > 3 - \frac{1}{x}.$$

Hint. Show that the function $f(x) = 2\sqrt{x} - 3 + 1/x$ is increasing on $[1, \infty)$.

Next Class: Thursday November 24

Optimization problems.
Limits at ∞ and L'Hospital's Rule.