MAT137 Lecture 2

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Agenda

- Negation.
- Double quantifiers.
- Simple proofs with quantifiers.

Girlfriends

Negate the following statement without using any negative words ("no", "not", "none", etc.):

"I have a friend all of whose siblings have at least two former girlfriends who had taken MAT137 twice."

Answer: "Each of my friends has a sibling that has at most one former girlfriend who had taken MAT137 twice."

Aliens

Let A be the set of aliens and H be the set of humans. For $x \in A$ and $y \in H$ let $E(x, y)$ denote "x eats y".

Translate the following statements into English

1. $\forall y \in H,\ \exists x \in A \quad$ such that $\quad E(x, y)$.
2. $\exists x \in A \quad$ such that $\quad \forall y \in H, \quad E(x, y)$.

Answer:

1. For every human there exists an alien that eats him/her.
2. There exists an alien that eats every human.

Quantifiers and the Empty Sets

Are the following statements true or false?

- There is a pink elephant in the room.
- All the elephants in the room are pink.

True or False?

If the statement is true, give a proof. If the statement is false, give a proof or a counterexample.

1. $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R}$ s.t. $y + \sin x > 0$.
2. $\exists y \in \mathbb{R}$ s.t. $\forall x \in \mathbb{R},\ y + x > 0$.

True or False?

$$\forall x \in \mathbb{R},\ \exists y \in \mathbb{R} \ \text{ s.t. } \ y + \sin x > 0.$$

- True.
- **Proof:** Pick any $x \in \mathbb{R}$, choose $y = 2$, then

$$y + \sin x > 2 + \sin x \geq 2 - 1 = 1 > 0,$$

as required. □

True or False?

$$\exists y \in \mathbb{R} \text{ s.t. } \forall x \in \mathbb{R},\ y + x > 0.$$

- False.
- **Proof:** To show that the statement is false, we show that its negation is true, i.e.

$$\forall y \in \mathbb{R},\ \exists x \in \mathbb{R},\ y + x \leq 0.$$

Pick any $y \in \mathbb{R}$, choose $x = -y$, then

$$y + x = y - y = 0 \leq 0,$$

as required. □

Dominance

Given two sets A and B of real numbers, we say that A **dominates** B if

$$\text{For every } b \in B \text{ there exists an } a \in A \text{ such that } a > b.$$

Find two non-empty sets A and B such that the following three properties are true

1. $A \cap B$ is empty,
2. A dominates B,
3. B dominates A.

Next Class: Thursday Sept 14

Watch videos 7, 8, 9, 10, 11, 12 ,13 in Playlist 1.