MAT137 Lecture 3

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Agenda

- Conditional Statements.
- Definitions and Proofs.

Some Negation Exercises

Write down the negations of the following statements as simply as you can:

1. There is a town in Ontario with fewer than 1000 inhabitants.
2. Every Canadian likes poutine.
3. There is a student in this class who does not have a facebook account.
4. If Peter does well on MAT137 then he does well on MAT237.
5. If Huan likes movies, then he likes pancakes too.
6. There is a washroom in every building at UofT.

Wonderland

I tell you

"If you get 90% or more on the first test, then I will take you to Canada's Wonderland."

In which of the following cases would my statement be a lie?

1. I take everyone to Canada's Wonderland.
2. You get 92% on your first test and I take you to Canada's Wonderland.
3. You get 60% on your first test and I take you to Canada's Wonderland.
4. You get 92% on your first test and I do not take you to Canada's Wonderland.
5. You get 60% on your first test and I do not take you to Canada's Wonderland.

Cards

Consider the following cards

A	B	5	8	O

Each card has a letter on one side and and a number on the other, and I tell you:

"If a card does not have a vowel on one side, then it has an odd number on the other side."

Which cards do you need to turn over in order to verify whether I am telling the truth or not? **Card B and card 8**.

Cards

Negate the following statement

"If a card does not have a vowel on one side, then it has an odd number on the other side."

Answer: "There is a card that does not have a vowel on one side and has an even number on the other side."

Bubble Tea

Which of the following statements are equivalent to the following statement:

"Every Asian girl likes bubble teas."

1. If a girl is not Asian, then she does not like bubble teas.
2. If a girl does not like bubble teas, then she is not Asian.
3. If a girl likes bubble teas, then she is Asian.
4. If a girl is Asian, then she likes bubble teas.
5. If an Asian does not like bubble teas, then that person is a boy.
6. There is at least one Asian girl who does not like bubble teas.
7. There is an Asian girl who likes bubble teas.

Absolute Values

Decide whether the following statement is true or false. Justify your answer.

$$\text{If } |x-1| < 2, \text{ then } |x-2| < 1.$$

- False.
- **Proof:** To show that the statement is false, we show that its negation is true, i.e.

  $$\text{There exists } x \in \mathbb{R} \text{ such that } |x-1| < 2 \text{ but } |x-2| \geq 1.$$

  We can just pick $x = 0$, then $|x-1| = |0-1| = 1 < 2$, but $|x-2| = |0-2| = 2 > 1$, as required. $\square$

A wrong proof

Discuss among yourselves what is wrong with the following argument:

Let $a = b$

Then

$a^2 = ab$

$a^2 + a^2 = a^2 + ab$

$2a^2 = a^2 + ab$

$2a^2 - 2ab = a^2 + ab - 2ab$

$2a^2 - 2ab = a^2 - ab$

$2(a^2 - ab) = a^2 - ab.$

Dividing both sides by $a^2 - ab$ gives

$2 = 1.$

Even and Odd

Write down formal definitions for what it means for an integer to be even or odd.

Which of the following is a correct definition for **odd**?

1. x is odd if $x = 2n + 1$.
2. x is odd if $\forall\ n \in \mathbb{Z},\ x = 2n + 1$.
3. x is odd if $\exists\ n \in \mathbb{Z},\ x = 2n + 1$.
4. x is odd if $\exists\ m \in \mathbb{Z},\ x = 2m - 1$.

Write down what it means by x is **even**.

Even and Odd

Discuss among yourselves why definitions **3** and **4** are equivalent to each other.

Even and Odd

Proof.

To show that **3** implies **4** we proceed as follows. Let x be odd. According to **3** there exists an integer n such that

$$x = 2n + 1.$$

Now we write

$$x = 2n + 1 = 2(n + 1) - 1.$$

Put $m = n + 1$, then m is an integer since n is. Therefore x has the form $2m - 1$ for some integer m, i.e. x is odd according to **4**. $\square$

Even and Odd

Proof.

To show that **4** implies **3** we proceed as follows. Let x be odd. According to **4** there exists an integer m such that

$$x = 2m - 1.$$

Now we write

$$x = 2m - 1 = 2(m - 1) + 1.$$

Put $n = m - 1$, then n is an integer since m is. Therefore x has the form $2n + 1$ for some integer n, i.e. x is odd according to **3**. □

Even and Odd

Consider the following theorem

Theorem

The sum of two odd integers is even.

Discuss among yourselves what are wrong with the following “proof”, and see you if you can improve it.

Proof.

$x = 2a + 1$
$y = 2b + 1$
$x + y = 2n$
$(2a + 1) + (2b + 1) = 2n$
$2(a + b + 1) = 2n$
$a + b + 1 = n.$ □

Even and Odd

A better proof:

Proof.

Let x and y be odd, then there exist integers a and b such that

$$x = 2a + 1 \quad \text{and} \quad y = 2b + 1.$$

Then

$$x + y = (2a + 1) + (2b + 1) = 2(a + b + 1).$$

If we denote $n = a + b + 1$, then n is an integer because a and b are integer. So $x + y = 2n$ for some integer n, i.e. $x + y$ is even. $\square$

All Real Numbers are Equal to 1??

Discuss among yourselves what are wrong with the following proof.

Proof(?)

Let $x \in \mathbb{R}$. Then

$$x^2 - 2x + 1 = 1 - 2x + x^2$$
$$(x-1)^2 = (1-x)^2.$$

Taking square root on both sides we get

$$x - 1 = 1 - x.$$

So

$$2x = 2.$$

Hence $x = 1$. $\square$

Next Class: Monday Sept 18

Watch videos 14, 15 in Playlist 1.