MAT137 Lecture 34

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Agenda

The divergence test.

The tail of a series

(a) Show that

$$\sum_{n=0}^{\infty} a_n \text{ converges} \iff \sum_{n=m}^{\infty} a_n \text{ converges}$$

for any positive integer m.

(b) Show that if $\sum_{n=0}^{\infty} a_n = L$, then $\sum_{n=m}^{\infty} a_n = L - \sum_{n=0}^{m-1} a_n$.

(c) Show that if $\sum_{n=m}^{\infty} a_n = M$, then $\sum_{n=0}^{\infty} a_n = M + \sum_{n=0}^{m-1} a_n$.

The tail of a series

Let $\sum_{n=0}^{\infty} a_n$ be a convergent series.

Let $R_n = \sum_{k=n+1}^{\infty} a_k$.

Show that

$$\lim_{n\to\infty} R_n = 0.$$

Examples

Evaluate the following series

(a) $\displaystyle\sum_{k=3}^{\infty} \frac{1}{k^2 - k}$.

(b) $\displaystyle\sum_{n=-2}^{\infty} \frac{(-1)^n}{5^n}$.

(c) $\displaystyle\sum_{n=1}^{100} 2^n + \sum_{n=101}^{\infty} \frac{1}{3^n}$.

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{2^{k-1}}{3^{2k+1}}$

The divergence test

Theorem

If the series $\sum_n a_n$ *is convergent, then* $\lim_{n\to\infty} a_n = 0$.

The divergence test

If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_n a_n$ is divergent.

Note. When $\lim_{n\to\infty} a_n = 0$, then NO conclusions can be made, the series can converge or diverge.

The divergence test

(a) Suppose that the series $\sum_n a_n$ is convergent and $a_n \neq 0$ for all n. Show that $\sum_n (1/a_n)$ diverges.

(b) Give an example of a series $\sum_n a_n$ such that $a_n > 0$ for all n, $\sum_n a_n$ diverges, but $\sum_n (1/a_n)$ converges.

(c) Give an example of a series $\sum_n a_n$ such that $a_n > 0$ for all n, $\sum_n a_n$ diverges, and $\sum_n (1/a_n)$ diverges.

Examples

Show that the following series diverge.

(a) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n^2+2}{3n^2+\sqrt{n}}\right)$.

(b) $\displaystyle\sum_{n=1}^{\infty} \arctan n$.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2}$.

(d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^n$.

(e) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$.

(f) $\displaystyle\sum_{n=2}^{\infty} \frac{n^{n-3}}{2n^4}$.

Next Class: Thursday March 8

Watch videos 13.10, 13.11, 13.12, 13.13, 13.14 in Playlist 13.