MAT137 Lecture 36

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Agenda

Absolute and conditional convergence.

The absolute convergence test

Theorem (The absolute convergence test)

If $\sum_{n}^{\infty} |a_n|$ *is convergent, then* $\sum_{n}^{\infty} a_n$ *is convergent.*

The converse is NOT true, a series that fails the converse is called conditionally convergent.

Absolute and conditional convergence

Definition

A series $\sum_{n}^{\infty} a_n$ is called

- **absolutely convergent** if $\sum_{n}^{\infty} |a_n|$ is convergent.
- **conditionally convergent** if $\sum_{n}^{\infty} a_n$ is convergent but $\sum_{n}^{\infty} |a_n|$ is divergent.

Give an example of an absolutely convergent series and an example of a conditionally convergent series.

Examples

Determine whether the series is absolutely convergent or conditionally convergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^{2/3}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{3n}}{n^2 + \ln n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(\ln n)}{3^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$

(e) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n+1)}$

Conditional convergence

- Show that if $\sum_{n}^{\infty} a_n$ is absolutely convergent, then $\sum_{n}^{\infty} a_n^2$ is convergent.
- Give an example of a conditionally convergent series $\sum_{n}^{\infty} a_n$ such that $\sum_{n}^{\infty} a_n^2$ converges.
- Give an example of a conditionally convergent series $\sum_{n}^{\infty} a_n$ such that $\sum_{n}^{\infty} a_n^2$ diverges.

Conditional convergence

- Suppose that $\sum_{n}^{\infty} a_n$ is conditionally convergent, show that $\sum_{n}^{\infty} n^2 a_n$ diverges.
- Give an example of a conditionally convergent series $\sum_{n}^{\infty} a_n$ such that $\sum_{n}^{\infty} n a_n$ converges.
- Give an example of a conditionally convergent series $\sum_{n}^{\infty} a_n$ such that $\sum_{n}^{\infty} n a_n$ diverges.

Next Class: Thursday March 15

Watch videos 13.18, 13.19 in Playlist 13.
Watch videos 14.1, 14.2 in Playlist 14.