MAT137 Lecture 40

Huan Vo

University of Toronto

March 26, 2018

Agenda

Computing Taylor series.

Multiplication of power series

Use the equation

$$\cos(x)\sec(x) = 1$$

and the Maclaurin series for $\cos x$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \cdots, \quad x \in \mathbb{R},$$

to find the Maclaurin series for $\sec x$ up to order 4. Since $\sec(x)$ is undefined at $\pm\pi/2$, the expansion is valid for $|x| < \pi/2$.

Taylor series

Exercise. Let

$$f(x) = \frac{1}{1+2x}.$$

Find the Taylor series for f centered at 1. What is the radius of convergence?

Taylor series

Exercise. Use the Maclaurin series for $\ln(1+x)$:

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots, \quad |x| < 1$$

to find the Maclaurin series for

$$f(x) = \ln(3 + x^2).$$

What is the radius of convergence?

Taylor series and compositions

Exercise. Let

$$f(x) = \ln(\cos x).$$

Find the Maclaurin series for f up to order 4. What is the radius of convergence?

Hint. Write

$$\ln(\cos x) = \ln(1 + (\cos x - 1)).$$

Then

$$\ln(\cos x) = -\frac{x^2}{2} - \frac{x^4}{12} - \cdots$$

The radius of convergence is $\pi/2$.

The sum of a series

Find the sum of the series

$$\sum_{n=0}^{\infty}(-1)^n\frac{x^{3n+2}}{n!} = 1 - x^5 + \frac{x^8}{2} - \frac{x^{11}}{6} + \cdots$$

Next Class: Thursday March 29

Watch videos 14.11, 14.12, 14.13, 14.14 in Playlist 14.