MAT137 Lecture 42

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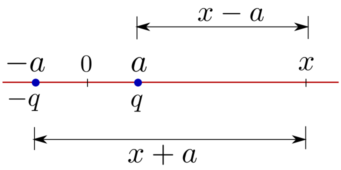
Agenda

Applications of Taylor series in physics.

Electric potential

- Suppose we have a charge with value q at the point $x = a$ and a charge with value $-q$ at the point $x = -a$. (This is called a **dipole**).



- The total **electric potential** at a point $x \gg a$ is given by

$$V = \frac{kq}{x - a} - \frac{kq}{x + a}$$

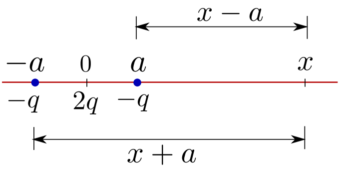
where k is a constant.

Electric potential

- Let $u = a/x$. Since $x \gg a$, we have $u \ll 1$. Write
$$V = \frac{kq}{x-a} - \frac{kq}{x+a}$$
as a Taylor series in u.
- Approximate V by the first non-zero term of its Taylor series. What conclusion can you make?

Electric potential

- Suppose that we have a charge with value $-q$ at $x = a$, a charge with value $2q$ at $x = 0$ and a charge with value $-q$ at $x = -a$.



- In this case the total electric potential at a point $x \gg a$ is given by

$$V = -\frac{kq}{x-a} + \frac{2kq}{x} - \frac{kq}{x+a}$$

where k is a constant.

Electric potential

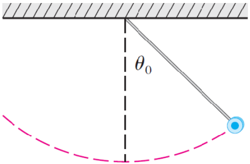
- Let $u = a/x$. Since $x \gg a$, we have $u \ll 1$. Write

$$V = -\frac{kq}{x-a} + \frac{2kq}{x} - \frac{kq}{x+a}$$

  as a Taylor series in u.
- Approximate V by the first non-zero term of its Taylor series. What conclusion can you make?

Pendulum

- Consider a pendulum with length L that makes a maximum angle θ_0 with the vertical



- It can be shown that the **period** T (the time for one complete swing) is given by

$$T = 4\sqrt{\frac{L}{g}} \int_0^{\pi/2} \frac{dx}{\sqrt{1 - k^2 \sin^2 x}}$$

where $k = \sin(\theta_0/2)$ and g is the acceleration due to gravity.

Pendulum

- Expand
$$T = 4\sqrt{\frac{L}{g}}\int_0^{\pi/2} \frac{dx}{\sqrt{1-k^2\sin^2 x}}$$
as a Taylor series in k. Recall that
$$\frac{1}{\sqrt{1-x}} = 1 + \frac{x}{2} + \frac{3x^2}{8} + \text{h.o.t.}, \quad |x| < 1,$$
and
$$\int_0^{\pi/2} \sin^{2n} x dx = \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots 2n}\frac{\pi}{2}.$$
- Approximate T by the first two non-zero terms of its Taylor series.

THANK YOU FOR YOUR ATTENTION!