MAT137 Lecture 5

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Agenda

- Inequalities and absolute values.
- The intuitive idea of limit.

Properties of Inequalities

Let $a, b, c \in \mathbb{R}$. Assume $a < b$, what can we conclude?

1. $a + c < b + c$
2. $a - c < b - c$
3. $ac < bc$
4. $a^2 < b^2$
5. $\frac{1}{a} > \frac{1}{b}$

Solving Inequalities

The solution set of

$$-2(x-3) > 4$$

is

1. $(-\infty, \infty)$
2. $(1, +\infty)$
3. $(-\infty, 1)$
4. $(-1, 1)$
5. $(-\infty, 1]$

Solving Inequalities

The solution set of

$$x^2 - 3x + 4 \leq 0$$

is

1. $(-\infty, \infty)$
2. $[-2, 2]$
3. $(-\infty, 2]$
4. $\emptyset$
5. who cares

Solving Inequalities

The solution set of

$$x(x+6) \geq 4+3x$$

is

1. $(-4, 1)$
2. $[-4, 1]$
3. $(-\infty, -4) \cup [1, +\infty)$
4. $(-\infty, -4] \cup (1, +\infty)$
5. $(-\infty, -4] \cup [1, +\infty)$

Properties of Absolute Values

Let $a, b \in \mathbb{R}$. What can we conclude?

1. $|ab| = |a||b|$
2. $|a + b| = |a| + |b|$
3. $|a| = 0$ if and only if $a = 0$
4. $|-a| = a$
5. $|a + b| \leq |a| + |b|$
6. $\big||a| - |b|\big| \leq |a - b|$

Inequalities and Absolute Values

The solution set of the inequality

$$|3 - 2x| > 1$$

is

1. $\emptyset$
2. $(1, 2)$
3. $(-\infty, 1) \cup (2, \infty)$
4. $(-\infty, 1)$
5. $(2, \infty)$

Sets Described by Distance

Let $a \in \mathbb{R}$ and $\delta > 0$. What are the following sets?

1. $A = \{x \in \mathbb{R} : |x| < \delta\}$
2. $B = \{X \in \mathbb{R} : |x| > \delta\}$
3. $C = \{x \in \mathbb{R} : |x - a| < \delta\}$
4. $D = \{x \in \mathbb{R} : 0 < |x - a| < \delta\}$

Implications

Find all values of α to make the following implication true

$$|x-2| < 1 \Longrightarrow |2x-4| < \alpha.$$

Answer: $\alpha \geq 2$.

Implications

Find all values of β to make the following implication true

$$|x - 2| < \beta \Longrightarrow |2x - 4| < 1.$$

Answer: $\beta \leq 1/2$.

Graphs

Let $a, L \in \mathbb{R}$. Let $\delta, \epsilon > 0$.

1. Draw the graph of a function f that satisfies

$$|x - a| < \delta \Rightarrow |f(x) - L| < \epsilon.$$

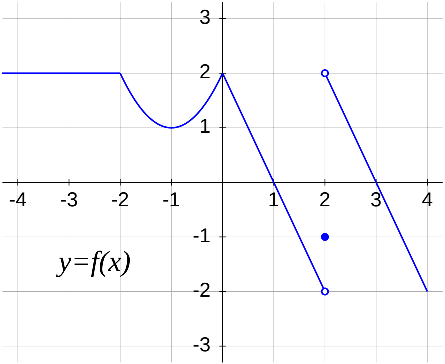
2. How does your answer change for...

$$0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon.$$

Find the smallest positive number M such that

$$|x-2|<1 \Longrightarrow |x+3|<M.$$

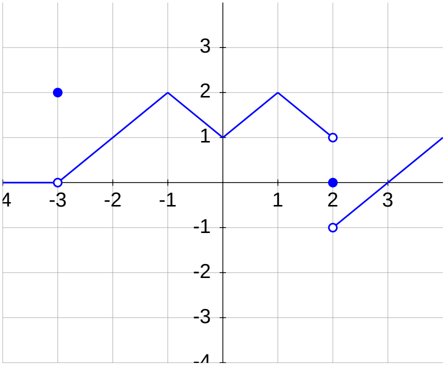
Limits from a graph



Find the value of

1. $\lim_{x \to 2} f(x)$
2. $\lim_{x \to 2^+} f(x)$
3. $\lim_{x \to 0} f(f(x))$
4. $\lim_{x \to 2} [f(x)]^2$
5. $\lim_{x \to 2} f(2 \sec x)$

Limits from a graph



Find the value of

1. $\lim_{x \to 2} f(x)$
2. $\lim_{x \to 1} f(f(x))$
3. $\lim_{x \to 2} f(f(x))$

Next Class: Monday Sept 25

Watch videos 5, 6, 7 in Playlist 2.