MAT137 Lecture 6

Huan Vo

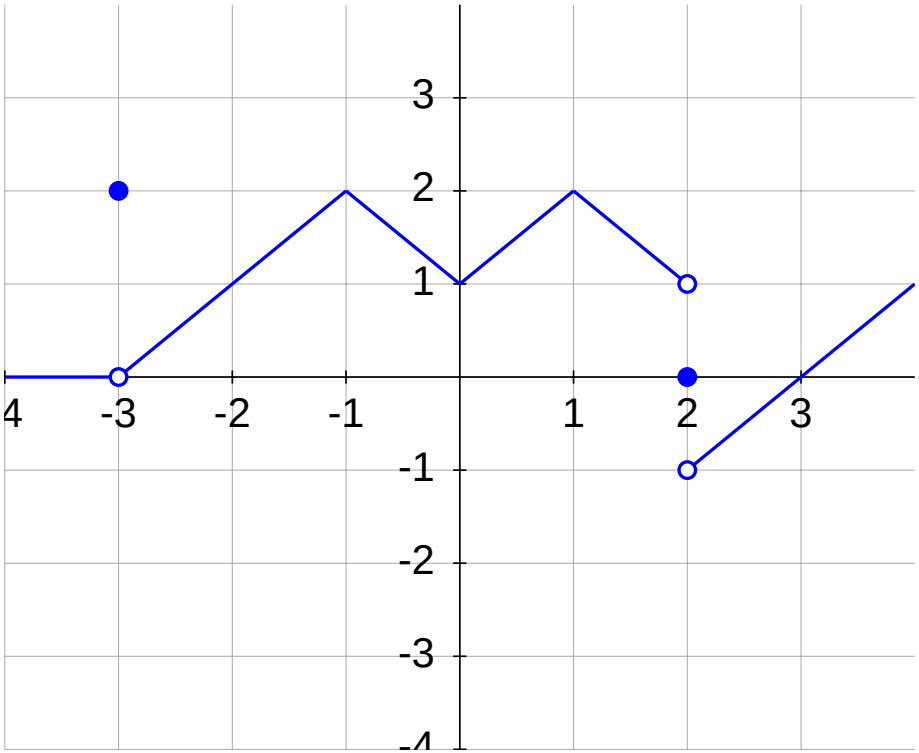
University of Toronto

September 25, 2017

Agenda

The formal definition of limit.

Limits from a graph



Find the value of

1. $\lim_{x \to 2} f(x)$
2. $\lim_{x \to 1} f(f(x))$
3. $\lim_{x \to 2} f(f(x))$

Side Limits

We know

Definition

Let $L, a \in \mathbb{R}$. Let f be a function defined at least on an interval around a, except possibly at a. Then

$$\lim_{x \to a} f(x) = L$$

means

$$\forall \varepsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Longrightarrow |f(x) - L| < \epsilon.$$

Write, instead, the formal definitions of

$$\lim_{x \to a^+} f(x) = L, \quad \lim_{x \to a^-} f(x) = L.$$

ε-δ proof

Prove, using the ε-δ definition of limit that

$$\lim_{x \to 2}(2x - 1) = 3.$$

Peer-Assisted Reflection - Feedback

Structure

1. Hypotheses and conclusion are clearly stated.
2. Introduce all the variables and notation.
3. Fix ϵ.
4. Clearly state δ.
5. The proof is self contained.
   (No need to read rough work to understand the proof)

Accuracy

1. Use correct definitions.
2. Computations are correct.
3. Right δ.

Clarity

1. It is clear what you are trying to prove.
2. Show all the steps.
3. Explain why, not just what.
4. Ideas follow logically one after another.

Presentation

1. Use complete English sentences.
2. Neatness.

ε-δ proof

This is a trickier problem. Prove, using the ϵ-δ definition of limit that

$$\lim_{x \to 2} \frac{1}{x+1} = \frac{1}{3}.$$

Analysis.

- Let $\varepsilon > 0$.
- $\left| \frac{1}{x+1} - \frac{1}{3} \right| = \left| \frac{2-x}{3(x+1)} \right| < \frac{\delta}{|3(x+1)|}$.
- Choose $|x-2| < 1$, then $|x+1| > 2$ (why?).
- Then $\left| \frac{2-x}{3(x+1)} \right| < \frac{\delta}{6}$. What δ should you choose?

□

Exercise: Can you write a formal proof of the above limit?

Next Class: Thursday Sept 28

Watch videos 8, 9, 10, 11, 12 in Playlist 2.