MAT137 Lecture 8

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Agenda

- The definition of continuity.
- Limits and composition of functions.

Definition of Continuity

- Let f be a function defined on an interval that contains a, write down the formal definition, i.e. using ε-δ, of the statement
  1. f is *continuous* at a.
  2. f is *continuous from the left* at a.
  3. f is *continuous from the right* at a.
- What does it mean to say that f is continuous on an interval $[c, d]$?

Continuity

Find all values of x so that the following function is continuous

$$f(x) = |x| + \sqrt{\frac{\log(1-x^2)}{(x-2)^3}}.$$

Compositions and Limits

What is wrong with the following proof?

Theorem??

Suppose that

1. $\lim_{x \to a} f(x) = b$,
2. $\lim_{u \to b} g(u) = L$.

Then $\lim_{x \to a} g(f(x)) = L$.

Compositions and Limits

Proof?

Let $\varepsilon > 0$ be given. Since $\lim_{u \to b} g(u) = L$ there exists $\delta > 0$ such that

$$0 < |u - b| < \delta \Longrightarrow |g(u) - L| < \varepsilon.$$

Since $\lim_{x \to a} f(x) = b$ there exists $\sigma > 0$ such that

$$0 < |x - a| < \sigma \Longrightarrow |f(x) - b| < \delta.$$

It then follows that if $0 < |x - a| < \sigma$, then $|f(x) - b| < \delta$, then $|g(f(x)) - L| < \varepsilon$. Thus $\lim_{x \to a} g(f(x)) = L$. □

Impossible Functions?

Can you find two functions f and g such that

1. $\lim_{x \to 1} f(x) = 2$,
2. $\lim_{u \to 2} g(u) = 3$,
3. $\lim_{x \to 1} g(f(x)) = 4$.

Compositions and Limits

Can you provide a proof of the following theorem?

Theorem

Suppose that

1. $\lim_{x \to a} f(x) = b$,
2. g *is continuous at* b

Then $\lim_{x \to a} g(f(x)) = g(b)$.

Next Class: Thursday Oct 5

Watch videos 16, 17, 18, 19, 20 in Playlist 2.