MAT137 Lecture 9

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Deadline to switch to MAT135: Friday October 6

Agenda

- Discontinuities.
- Trigonometric Limits.
- Extreme Value Theorem (EVT).
- Intermediate Value Theorem (IVT).

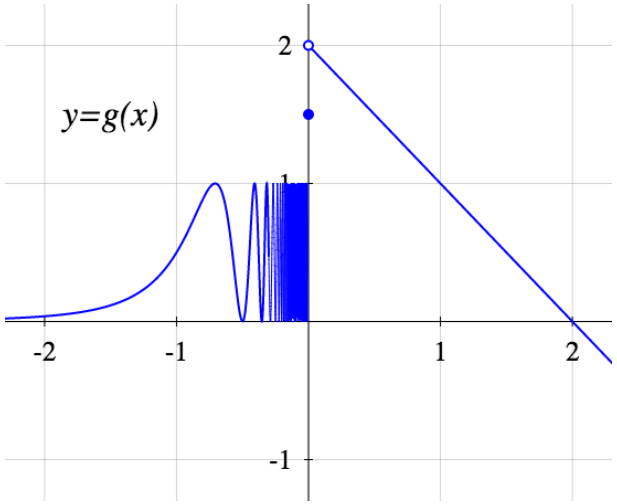
Floor

Given a real number x, we defined the *floor of* x, denoted by $\lfloor x \rfloor$, as the largest integer smaller than or equal to x. For example, $\lfloor \pi \rfloor = 3$, $\lfloor 7 \rfloor = 7$, and $\lfloor -0.5 \rfloor = -1$.
Compute

1. $\lim_{x \to 0^+} \lfloor x \rfloor$
2. $\lim_{x \to 0^-} \lfloor x \rfloor$
3. $\lim_{x \to 0} \lfloor x \rfloor$
4. $\lim_{x \to 0} \lfloor x^2 \rfloor$

More limits from a graph

Below is the graph of the function g:



Compute

1. $\lim_{x\to 0^+} g(x)$
2. $\lim_{x\to 0^+} \lfloor g(x) \rfloor$
3. $\lim_{x\to 0^+} g(\lfloor x \rfloor)$
4. $\lim_{x\to 0^-} g(x)$
5. $\lim_{x\to 0^-} \lfloor g(x) \rfloor$
6. $\lim_{x\to 0^-} \lfloor \frac{g(x)}{2} \rfloor$
7. $\lim_{x\to 0^-} g(\lfloor x \rfloor)$

A function discontinuous everywhere

Show that the following function

$$f(x) = \begin{cases} 2 & \text{if } x \in \mathbb{Q}, \\ -2 & \text{if } x \notin \mathbb{Q} \end{cases}$$

is not continuous anywhere on $\mathbb{R}$.

Discontinuities

Let

$$f(x) = \begin{cases} A^2x^2, & x \leq 2, \\ (1-A)x, & x > 2. \end{cases}$$

For what values of A is f continuous at 2?

Trigonometric Limits

Evaluate

$$\lim_{x \to 0} \frac{x^5 \sin(3x)}{\sin^2(x^3)\cos(x^2+1)}.$$

Trigonometric Limits

Evaluate

$$\lim_{x \to 0} \frac{1 - \cos(2x^3)}{x^6}.$$

Next Class: Thursday Oct 12

Watch videos 2.19, 2.20, 3.1, 3.2, 3.3, 3.4 in Playlist 3.